Learning to write about Mathematics

This student-friendly rubric helped improve third graders' competencies when explaining solution strategies in writing.

By Renee Parker and M. Lynn Breyfogle

Beginning in third grade, Pennsylvania students are required to take the Pennsylvania State Standardized Assessment (PSSA), which presents multiple-choice mathematics questions and open-ended mathematics problems. Consistent with the Communication Standard of the National Council of Teachers of Mathematics (NCTM 2000), while solving the open-ended problems, students must explain and describe the process they follow to arrive at their answer. In recent years, Renee Parker, a third-grade teacher in Central Pennsylvania, found that many of her students were able to determine correct answers to the open-ended mathematical problems. However, they would not receive full credit for their responses because they could not clearly communicate their thinking about a problem. Parker realized that her students needed to improve their ability to write about their processes and mathematical thinking.

During the 2006–2007 school year, Parker participated in a year-long professional development project, Math Proficiency for All (MathPro) led by the co-author, a university professor. MathPro was created to help kindergarten through grade 6 teachers attend to the mathematical thinking of their students. As part of the professional development experience, each teacher was expected to engage in an action research project designed to improve his or her mathematical instruction. Parker and two colleagues who also teach third grade chose to focus on improving third graders' problem-solving processes by encouraging them to write about their thinking. The teachers thought that creating an ongoing formative assessment strategy using a rubric might help students. What follows is Parker's story about her action research project and what she learned about using a math rubric with elementary school children.

Parker's story

In the past, my colleagues and I had students explain their mathematical thinking through

FIGURE 1

Each of five weekly problems in an identical format required students to draw a diagram, write a number sentence, and explain their process as well as their steps to find the answer.

Problem 1	Beth and Jim brought some pencils to school. Beth brought seven pencils, and Jim brought six pencils. How many pencils did they bring to school in all?
Problem 2	Sam has 12 books about sports. His brother, Joe, has 5 books about sports. How many more books about sports does Sam have than Joe?
Problem 3	Jenny takes six dogs for a walk. After the walk, she gives each dog two treats. How many treats does Jenny give to the dogs in all?
Problem 4	There are five groups in Mr. Smith's art class. He gives each group three crayons to use for a project. How many crayons did he give to his students?
Problem 5	Alex has 12 candies. He is going to give an equal number of candies to each of his four friends. He is not keeping any of the candies for himself. How many candies will each friend get?

writing, but never in a systematic way. For this project, I wanted to offer a way for students to reflect on their own writing to improve their thought processes, problem-solving skills, and mathematical writing. The following question guided my investigation: Does the use of a student-friendly rubric (SFR) to assess students' writing improve their ability to communicate their thinking while problem solving?

The process

To investigate this question, my colleagues and I created and used a total of five problem-solving questions over a five-week period. The problems were based on the Pennsylvania Assessment Standards and Anchors assessed through the Pennsylvania State Standardized Assessment. Our students solved each of the five problems, one problem each week (see **fig. 1**). Each problem was presented in the same format with the intention of helping the children demonstrate their thinking and encouraging their use of multiple representations.

When deciding on an effective rubric to use for the research, my colleagues and I talked about what elements are important to improve students' writing and thinking. We decided we wanted a rubric that assessed three specific elements:

1. Understanding of mathematical concepts
2. Planning and using strategies to solve a problem
3. Explaining mathematical actions and thinking through writing

My colleagues and I adapted a "student-friendly math rubric" (Illinois State Board of Education 2009) that we found online. To make the rubric more user-friendly, I felt that some of the language had to be simplified and some parts of the rubric should be shifted and included in other areas. For instance, I replaced the pronoun *it* with the words *the problem* for clarification and the phrases *do the problem* and *work it out* several times with *solve the problem* to keep students focused on the problem-solving process. I included more action verbs and changed the wording to match the prompts and words that I use in my daily math instruction. Furthermore, I used boldfaced type and underlining to highlight mathematical language in the rubric to help students focus on the mathematical language and explanations in their writing. To check that they were meeting their teacher's expectations, students used the adapted rubric (see **table 1** in the **online** appendix) while completing the problem-solving tasks. Meanwhile, my colleagues and I used the rubric to evaluate and score students' writing, mathematical understanding, and problem solving. Before third grade, many students are unfamiliar with rubrics. Beginning in third grade, students have

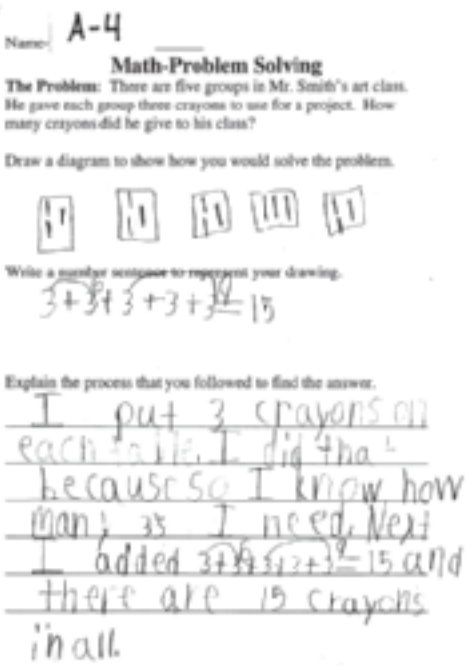
experience using rubrics as an assessment tool in other subjects, but this was an introduction to using a rubric in mathematics class.

For the research project, all students in the class participated, but I closely monitored the performance of three predetermined students, purposely chosen on the basis of their previous mathematical performance in my classroom. I followed one student from each of the three ability groups, below average (BA), average (A) and above average (AA). As an initial assessment, the first problem and student-friendly rubric (SFR) were presented to the students without any formal instruction or prompting. I simply read the question and had students complete the three parts of the problem-solving task. After collecting their work, I independently scored each student's work with the SFR. During the following week, students were given the second problem-solving task, and we discussed the SFR briefly as a class.

After each of weeks two, three, and four, the students and I reviewed samples of several students' work to become more familiar with the SFR and improve their own writing and problem solving. As a class, we used the rubric to grade three anonymous pieces of student work. To give students the opportunity to see samples of quality work that met the grading expectations, I chose work that met or nearly met the rubric requirements. Then, following week four, I shared two anonymous work samples from the average student. As a class, we compared the student's previous work sample from week one to current work from week four (see **fig. 2**). I began the conversation by asking students to determine what they thought the current work would earn on the rubric. Then we looked closely at each of the rubric categories. The following conversation is an excerpt from a group discussion that focused on "using problem-solving strategies." My comments are in bold type.

FIGURE 2

Following week four, the teacher presented two anonymous work samples from an average student for the class to compare. Below is the work from week four.


Name: A-4

Math-Problem Solving

The Problem: There are five groups in Mr. Smith's art class. He gave each group three crayons to use for a project. How many crayons did he give to his class?

Draw a diagram to show how you would solve the problem.

Write a number sentence to represent your drawing.

3+3+3+3+3=15

Explain the process that you followed to find the answer.

I put 3 crayons on each table. I did that because so I know how many is I need. Next I added 3+3+3+3+3=15 and there are 15 crayons in all.

OK, let's look at the student's problem-solving strategies. Was all of the important information from the problem used?
[*Student 4*] I think so. The important information is the number of groups and the number of crayons each group has. There are five groups, and each group has three crayons. He used the three and the five to solve the problem.

Great! I like how you were able to find the important information in the problem. Does everyone think the student showed all the steps that were used to solve the problem?
[*Student 1*] Yes. He drew boxes to show the groups and three tally marks in each box to show the crayons.

So, the student used a diagram.
[*Student 4*] He drew a picture, but he didn't make a key.

You're right. I think that's OK, but a key is always a good idea. It helps to explain our work. Let's look at the number sentence. Who can talk to me about the number sentence?
[*Student 5*] He used repeated addition. He added the groups of three.

I see the repeated addition. That is a great strategy to use when we are adding groups of equal size. Does anyone notice anything else about the number sentence?
[*Student 3*] He grouped the first two threes together to make six and the next three threes together to make nine. Then he added six plus nine to get fifteen.

Oh, I see. How else do you think you could have added those five threes?

FIGURE 3

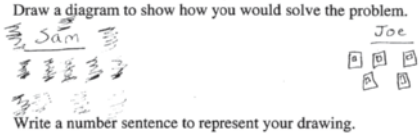
Discussing the rubric helped the above-average student to use mathematical terminology and explain her thinking.

Name- AA-2

Math-Problem Solving

The Problem: Sam has 12 books about sports. His brother, Joe, has 5 books about sports? How many more books about sports does ~~Joe~~ Sam have?

Draw a diagram to show how you would solve the problem.



Write a number sentence to represent your drawing.

12-5=7

Explain the process that you followed to find the answer.

Well Sam had 12 books his brother Joe had 5 books. So I made a subtraction problem. I subtracted 5 from 12 and I found the sum useing the addends.

[*Student 5*] He could have skip counted by threes five times to get fifteen. He could also multiply.

Great observation! The number sentence is correct, but other number sentences could have been used, too. Did anyone use a different number sentence to solve the problem?
[*Student 4*] I used multiplication. Multiplying three times five or five times three would be OK, too.

In addition to the group conferences, students met with me individually each week to exchange ideas about their own work and ask me questions about how to improve their writing. Unlike the whole-group conference, which focused on modeling a good but not perfect example, the individual conference had students evaluate their own work. I walked each student through the rubric and asked him or her to orally evaluate the work sample. Then I added my suggestions and comments. Whole-group conferences helped students to better understand the rubric; individual conferences allowed students to focus more closely on their own work and identify problems they were having that were not necessarily present in the work samples we reviewed as a class. Both types of conferences helped students become more familiar with math language, expectations, and math thinking. The following excerpt from the individual conference with student AA is based on the work shown in **figure 3**:

Yes. Well, what about the last category, Writing an Explanation?
I think that's a five.

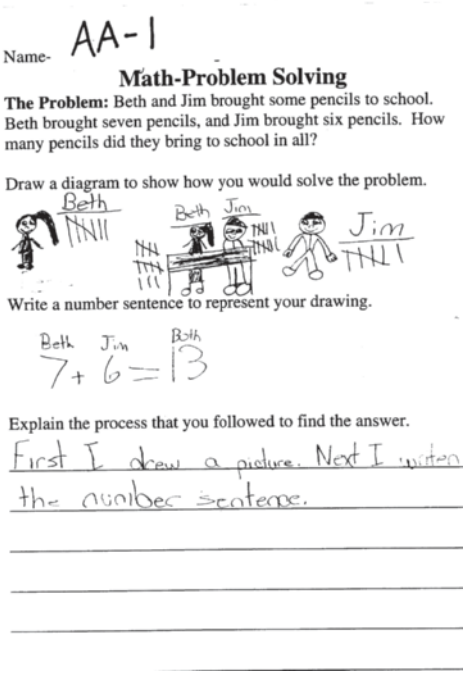
Because?
I wrote what I did, and I used math words. I wrote *sum* and *addends* and subtracted.

Let's look at the first bullet. You wrote what you did, but did you say why you did it?
No [*pausing*]. I just wrote what I did. I didn't say why.

How about "explain each step of my work?" I'm a little confused with what you say. In this sentence, you talk about "subtraction," and then in the next sentence, you talk about "addends and sums." Can you say what you were thinking?

FIGURE 4

In her first task, the same student had explained her process as a list of what she did. She talked about the *what* but not the *why* of her work.


Name- AA-1

Math-Problem Solving

The Problem: Beth and Jim brought some pencils to school. Beth brought seven pencils, and Jim brought six pencils. How many pencils did they bring to school in all?

Draw a diagram to show how you would solve the problem.

Beth Jim Beth Jim

Write a number sentence to represent your drawing.

Beth Jim Both
7 + 6 = 13

Explain the process that you followed to find the answer.

First I drew a picture. Next I writen the number sentence.

When I read the problem, I knew it was subtraction. That's why I wrote 12 – 5 = 7, but I didn't think about it that way. I just started with five and knew five plus seven was twelve. And you said last week to try to use more math words, so I wanted to use them on my paper.

Yes, I see; and I'm very happy that you are using more of the math words—that's great! But, what we really want to do is make sure that what we write is really the way we think about the problem. And also that the drawing and number sentence all show the same thing. So, let's talk about how it would look if we showed the way you thought about it.

As AA's work shows, students' number sentences or pictures answered the problem, so both were not necessary. However, I wanted students to become familiar with showing a single problem in various ways. Also, some students prefer one strategy over another. Being required to supply both a picture and a number sentence gave students the chance to improve both skills.

The individual and group discussions allowed students to better understand the problems, expectations, and mathematical concepts. As students became more familiar with the expectations and problem-solving tasks, the rubric began to serve as a guide in helping them focus on the three elements that I assessed. By the end of the five weeks, each student in the sample scored at least fourteen of fifteen possible points using the student-friendly rubric. The area where I felt students made the most progress was in the explanation of the steps they followed to solve the problem. Although this area continued to be where students lost points, I felt this was where they developed the most as mathematical writers. Moreover, students strengthened their ability to incorporate mathematical vocabulary and strategies into their writing. Not surprisingly, the students of below-average and average ability showed the greatest improvement in these areas.

Role of the rubric

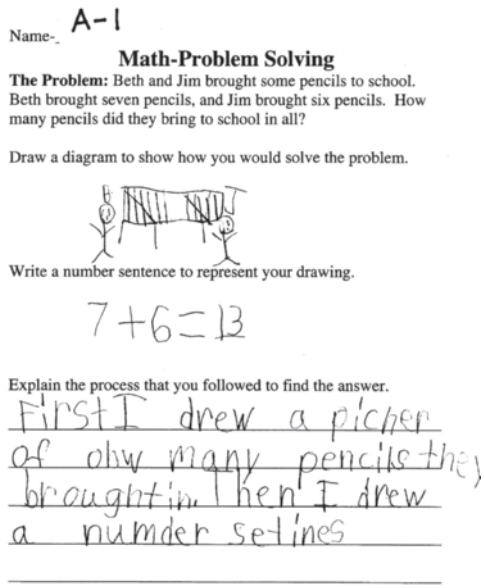
Discussing the rubric helped the above-average student to more thoroughly explain her thinking, but the discussion was more about meeting expectations than it was about developing her ability to verbalize her thinking. In her first task (see **fig. 4**), she had explained her process as a list of what she did: "First I drew the picture. Next I writen [wrote] the number sentence." She had earned thirteen points for her work, losing two points on Writing an Explanation. She talked about the *what* but not the *why* of her work. Whereas once the rubric was described, in her second task (see **fig. 3**), she included the mathematical terminology (albeit not all correct) and described more clearly how she was thinking: "So I made a subtraction problem. I subtracted five from twelve, and I found the sum [by] using the addends." She still earned a total of thirteen points. Her work showed a better understanding of Writing an Explanation, but she still needed to expand on why she solved the problem in the way that she did.

For the average student, the rubric did little to change the explanations between the first task (see **fig. 5a**) and the second task (see **fig. 5b**). He was able to draw a diagram and write a number sentence. His explanations tell what he did but

FIGURE 5

The rubric did little to improve the average student's ability to explain his thinking between—

(a) the first task and



Name- A-1

Math-Problem Solving

The Problem: Beth and Jim brought some pencils to school. Beth brought seven pencils, and Jim brought six pencils. How many pencils did they bring to school in all?

Draw a diagram to show how you would solve the problem.

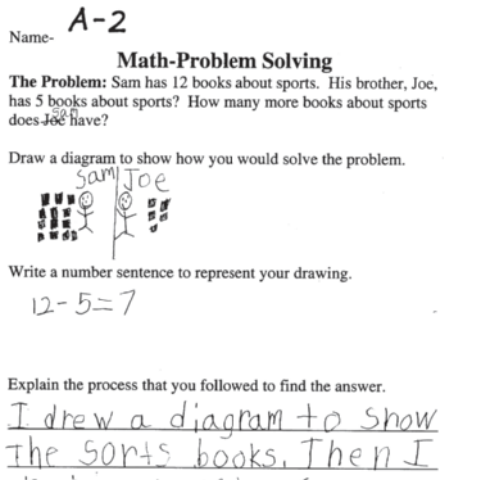
Write a number sentence to represent your drawing.

7+6=13

Explain the process that you followed to find the answer.

First I drew a picher of ohw many pencils they brought in. Then I drew a numder setines

(b) the second task.



Name- A-2

Math-Problem Solving

The Problem: Sam has 12 books about sports. His brother, Joe, has 5 books about sports? How many more books about sports does ~~Joe~~ Sam have?

Draw a diagram to show how you would solve the problem.

Sam Joe

Write a number sentence to represent your drawing.

12-5=7

Explain the process that you followed to find the answer.

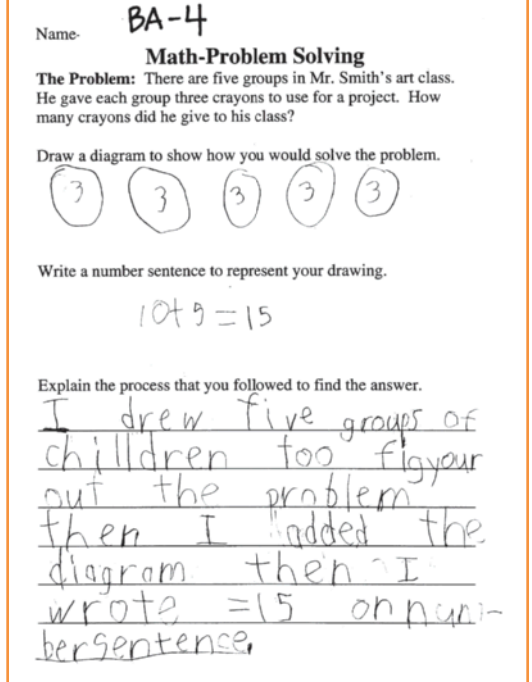
I drew a diagram to show the sorts books. Then I rout a numder sentence to show how many more books Sam has then Joe,

do not include an explanation of why he solved the problem in that way. The other two students' work was similar in this respect. On both tasks, the students earned a total of thirteen points because they did not include details about their mathematical thinking. Points were lost in the Writing an Explanation portion of the rubric.

What seemed more helpful was the inclusion of individual conferences with the students to explicitly discuss their work using the rubric. I saw a dramatic difference in the explanations with the fourth problem-solving task for both the below-average and average students. The average student's explanation now included a description of adding the number of threes. He then continued his description to include exactly how (grouping marks above the threes into six and nine) he had added the crayons to determine the answer of fifteen. Prior to this fourth problem-solving task, he had recorded only such statements as, "Then I did a number

FIGURE 6

Although the below-average student's number sentence does not represent his drawing, by the fourth task, he began to include some mathematical language to explain his thought processes.



Name- BA-4

Math-Problem Solving

The Problem: There are five groups in Mr. Smith's art class. He gave each group three crayons to use for a project. How many crayons did he give to his class?

Draw a diagram to show how you would solve the problem.

3 3 3 3 3

Write a number sentence to represent your drawing.

10+9=15

Explain the process that you followed to find the answer.

I drew five groups of chilldren too figyour out the problem then I added the diagram then I wrote =15 on numbersentense.

sentens [sentence] to tell how many bowns [bones] there where [were].” This student earned fourteen points on the fourth problem-solving task. He lost one point for Writing an Explanation because he did not thoroughly explain why he solved the problem in this way.

The below-average student demonstrated a similar pattern of improvement. He began to show evidence of using mathematical language in his explanation. Comparing his second task with his third task shows a more elaborate explanation: “I drew dog treats so I could sovle [solve] the problem and a number sentence. So I could [count] how many dog-treats were left,” opposed to, “I youwhile [used] talle [tally] marks and a number sentece [sentence].” In both, he described the diagrams but not how he used them. He also referenced the number sentence but not how it related to the diagram. The below-average student’s fourth task (see **fig. 6**) shows that he began to include some mathematical language—“then I added the diagram”—to explain his thought processes. The student earned ten points for his work, including four points for Showing Math Knowledge, three points for Using Problem-Solving Strategies, and three points for Writing an Explanation. Not until this student’s fifth task did we begin to see descriptions of his mathematical thinking rather than his process. This student earned a total of fourteen points, losing only one point

FIGURE 7

Not until the below-average student's fifth task did we begin to see descriptions of his mathematical thinking rather than his process.

Name- BA-5

Math-Problem Solving

The Problem: Alex has 12 candies. He is going to give an equal number of candies to each of his four friends. He is not keeping any of the candies for himself. How many candies will each friend get?

Draw a diagram to show how you would solve the problem.

Write a number sentence to represent your drawing.

12 -3-3-3-3=0

Explain the process that you followed to find the answer.

I drew three tally-marks four each friend. And then I subtracted 12-3-3-3-3. each friend will get 3 candies.

for Writing an Explanation because, although he touched briefly on the *why* (see **fig. 7**), he could have added more details about his mathematical thinking. We might not completely understand how he came up with an answer of three—his drawing suggests that he used a partitive approach to the division problem, and his explanation suggests a measurement approach because of his repeated subtraction—but we do see a shift toward recording his thinking. Although we had discussed the SFR before week two, it seemed that a necessary piece in helping the average and below-average students to use the rubric was to explicitly point out—in their own work and in exemplary models—the important aspects to include in an explanation.

Lessons learned

On the basis of my action-research project, I believe that explicitly teaching students to use an SFR to evaluate mathematical writing improves students' ability to write about mathematics and develops their skills as independent learners and problem solvers. During the five-week period of my study, the students were able to develop more precise explanations of the problem-solving process and include reasons why they solved the given problems in a certain way. Students began incorporating more appropriate mathematics vocabulary into their writing and demonstrating competency in deciding what each problem was asking them to do. Finally, all the regular education students in my class rated proficient or advanced on the PSSA mathematics test in the spring of 2007, an increase of over 30 percent from my previous year's third-grade students. By using an SFR, having opportunities to write about their thinking, and critiquing this writing, students improved their understanding of mathematical concepts and writing skills as well as decreased their anxiety about taking the PSSA.

Since the 2007–2008 school year, I have used various strategies to encourage students to write about mathematics on a more regular basis. The success that I observed when students completed the problem-solving tasks with the SFR prompted me to continue this writing exercise

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and include other ones in my mathematics instruction. For instance, on certain days, my students complete a problem-solving task and write about it using a vocabulary box, which includes words that may be helpful in explaining their thinking, such as *added*, *addends*, *sum*, and *equals*. Students must complete the task and include a predetermined number of words from the vocabulary box in their explanation. Incorporating vocabulary activities in mathematics writing helps students become more familiar with mathematical language that they hear and see each day. This project helped me recognize not only the importance of engaging students in writing about mathematics to help them understand the mathematics but also how their writing supplies me with insight into their thinking and informs my own teaching.

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Go to **www.nctm.org/tcm** to access **table 1** in the **appendix**, which accompanies the online version of this article.